"Differential and Integral Methods"

(before 2016)

1. Real-valued functions, the domain, the range, graphs, shifting graphs, increasing and decreasing, inverse functions, composite functions.
2. Elementary functions: linear and quadratic, polynomials, power, exponential, logarithmic, trigonometric, hyperbolic, absolute value, integer.
3. Informal definition of limit of functions, continuous functions. Number e as a limit, the limit of Sin(x) divided by x. Continuity of a function using sequences and using epsilon-delta, one-sided limits and continuity, the intermediate value theorem, inverse function and its continuity. Existence of extremum. Continuity of elementary functions.
4. Derivative as a tangent slope and a velocity, tangent and normal lines to functions. Calculating derivatives of polynomials, negative powers, Sin(x), Cos(x). Differentiation rules, derivative of tan(x) and inverse functions.
5. The chain rule, derivative of rational powers, derivatives of sinh(x), cosh(x), tanh(x), arcsinh(x), arccosh(x), arctanh(x). Derivative of a in power x using the chain rule. Parametrizations of plain curves and their derivatives.
6. Rolle theorem, the intermediate value theorems of Lagrange and Cauchy.
7. Linearization and differentials. Taylor's formula with a remainder and Taylor series, the proof of Taylor formula with Lagrange remainder. Taylor's formula of elementary functions. Application to l'Hopital's rule. Application of Taylor series to binomial series. Application of Taylor's formula to sufficient condition of an extremum. Investigation of a function.
8. Complex numbers, Euler's formula, complex representation of trigonometric functions.
9. Indefinite integral, integral formulas, definite integral and area, Darboux integrals. The fundamental theorem of calculus, evaluating integrals. Substitution, integral of rational functions, integration by parts, trigonometric substitutions, improper integral. Integrals which depend on a parameter and their derivative with respect to the parameter (Leibniz's rule). Evaluating integrals using series.

10. Applications of integrals: area between curves, the length of curves, volumes of solids of revolution, moments and centers of mass.
11. Limit and continuity of functions of two variables, partial derivatives, gradient, tangent and normal planes to surface. The chain rule, differentials, implicit differentiation. Taylor's formula for functions of two variables. Extremum. Lagrange multiplier method.
12. Double and triple integrals, iterated integrals.
13. Line integral of scalar functions. Line integral of vector-functions. Work. Path independent line integrals (conservative fields). Green's theorem (in the plane).
14. Surface area and surface integrals. Theorems of Stokes and Gauss.

Books:

- Ben Zion Kun and Sami Zafrani, "Heshbon Diferenziali ve Integrali 1 ve 2", BAK, Haifa, 2000 (in Hebrew).
- Thomas and Finney,"Calculus and Analytic Geometry", 9-th edition, Addison-Wesley, 1996.
- Arfken and Weber,"Mathematical Methods for Physicists", 4-th edition, Academic Press, 1995.

"Calculus 1b"

(after 2016)

COURSE DESCRIPTION

We are going to investigate real-valued functions of a single variable. That includes, in particular, limits, differentiation and integration of the functions, investigation of their extremum, approximation of the functions by polynomials. But, first, we start with numerical sequences and series and conclude the course with sequences and series of functions of a single variable.

COURSE TOPICS

- Topics from the set theory. Infinite sequences. Limit of sequences, divergence, monotonic sequences, the sandwich theorem, subsequences, Bolzano-Weierstrass theorem. Cauchy characterization of convergence. Infinite series, convergence and divergence of series, convergence tests of series. Absolute and conditional convergence.

- Real-valued functions, increasing and decreasing functions, inverse functions, composition of functions. Elementary functions: linear and quadratic, polynomials,

power, exponential, logarithmic, trigonometric and their inverse, hyperbolic, absolute value, floor function. Informal definition of limit of functions and continuity - using sequences and epsilon-delta, one-sided limits and continuity. The intermediate value theorem, Weierstrass theorem.

- Uniform continuity. The squeeze theorem. Number e as a limit, the limit of Sin(x) divided by x. Derivative as a tangent slope and a velocity, tangent and normal lines to functions. Calculating derivatives of polynomials, negative powers, Sin(x), Cos(x). Differentiation rules, derivative of tan(x) and inverse functions. The chain rule, derivative of rational powers, derivatives of sinh(x), cosh(x), tanh(x). Derivative of a in power x using the chain rule. The mean value theorems of Rolle and Langrange.

- Linearization and differential. Taylor's formula with a remainder and Taylor series, the proof of Taylor's formula with Lagrange remainder. Taylor's formula of elementary functions. Its application to l'Hopital's rule and to sufficient condition of an extremum. Convexity and inflection points. Asymptotes. Investigation of a function.

- Indefinite integral, integral formulas: substitutions, integral of rational functions, integration by parts. Definite integral and area. The fundamental theorem of calculus. Integrals which depend on a parameter and their derivative with respect to the parameter. Applications of integrals: area between curves, the length of curves, volumes of solids of revolution, moments and centers of mass. Improper integrals.

REQUIRED READING

Protter and Morrey, *A first Course in Real Analysis*, 2nd edition, Springer, 1991.

ADDITIONAL READING

Thomas and Finney: *Calculus and Analytic Geometry*, 9th edition, Addison-Wesley, 1996.
Arfken and Weber, *Mathematical Methods for Physicists*, 4th edition, Academic Press, 1995.
Any other book in calculus (for engineering faculties and higher) can be used.